On Population Dynamics

Collin Anderson

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The idea of predicting the future growth and change in a population of a species by previous trends is nothing new. There is some intrinsic knowledge that man has always had concerning the nature of population levels and growth. We have always known to some extent that a population over hunted will die out forever, and a population that has little competition or opposing predators will overpopulate and become weaker as natural resources become too thinly spread.

Species can encounter such an enormous variation of difficulties that complete volumes of books can be written just for each different condition. For an overpopulated group, that lack of food can be made worse by the likely ensuing epidemic. For a grossly underpopulated species, birth rates can be reduced by the fact that a male cannot even find a mate within its locale. This complexity greatly increases the higher you travel along evolution's path. Humans have unique constructs like war, religion and politics which are even less predictable.

What is new to the history of man is the efforts to build an empirical structure to this math and apply it to assess the growth of organisms. Overall, there are only two basic variables with determine the rate of growth of the population, the number of births and the number of deaths, relevant in rates per one thousand. In general, we use the general term r to measure the difference between birth and death rates. A growing population will have a positive r and a negative r will determine a dying population. This simple logic can be thought of the most basic, elementary population model. The farther we step, the more complexity we add, for example, two more factors become important when we are measuring an artificial distinction in a population, most commonly a nation-state, emigration and immigration.

The origin of the story of demographics and population modeling is widely accredited to the 17^{th} century political scientist and economist Thomas Malthus. As a response to the views of William

Godwin, Jean-Jacques Rousseau, David Hume and his father, Malthus published the landmark work *Essay on the Principle of Population* [4], asserting not only a geometric population growth, but a real, human importance and relevance to what would seem like a pure statistical science. There is an remarkability in the method in which Malthus injects mathematics into philosophy.

Malthusian Geometric Growth Equation 1 *Where x_n is the base measurement of a population, r is the growth rate of the population (birth rate - death rate)*

$$x_{n+1} = rx_n(1 - x_n)$$

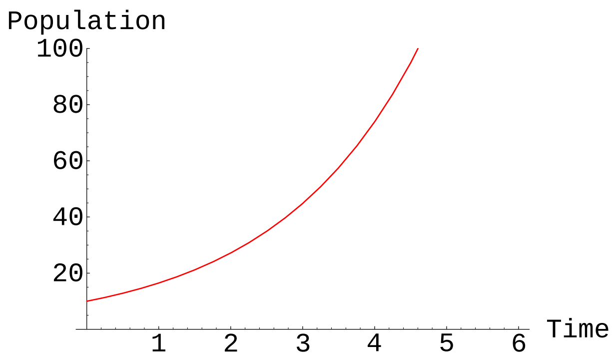

Figure 1: Malthusian Exponential Growth Equation

Despite its mathematical significance, *Principle of Population* was in fact mostly a combination of religious and political propaganda and philosophical discussion. Malthus argues that the policy of protecting the poor that was the inspiration of laws such a the Poor Laws of England and the vision of Hume is impossible because of nature. His essay first establishes a famous axiom.

> *Population, when unchecked, increases in a geometrical ratio. Subsistence increases only in an arithmetical ratio. A slight acquaintance with numbers will show the immensity of the first power in comparison of the second.*

From which Malthus proclaims that the growth rate of the poor must balanced by a high mortality rate among the lower class or else the overall quality of life of the poor will decrease as a result of the stress of spreading already low resources to larger and larger segments of the population.

Aside from earning the label of inventing the *"the crudest, most barbarous theory that ever existed"* from Friedrich Engels as a result of its use in enforcing the morality of class indifference, Malthus used his theory to advocate sexual morality. Throughout the essay, the author uses numerous case studies of situations such as indigenous people to show how changes in marriage ages can affect the welfare of the society. It is ironically humorous that today Malthus's apocalyptic visions of overpopulation are being used to promote birth control despite the fact that he was virulently anti-contraceptive

In the end, Thomas Malthus's model for growth of a population assumes an ideal environment, where resources and growth are not artificially or naturally limited by any third party. The population increases at a rate proportional to the number of individuals present resulting in an exponential growth curve.

> *The power of population is so superior to the power of the earth to produce subsistence for man, that premature death must in some shape or other visit the human race...But should they fail in this war of extermination, sickly seasons, epidemics, pestilence, and plague, advance in terrific array, and sweep off their thousands and tens of thousands. Should success be still incomplete, gigantic inevitable famine stalks in the rear, and with one mighty blow levels the population with the food of the world.* [4]

The fundamental peculiarity of Malthus's work is that despite his emphasis on population limits, he never factored in the limits of the growth of population by environment into his equation. It wasn't until Pierre François Verhulst developed the logistic demographic model after reading Malthus's work that populations had physical limits to growth. Like the Malthus model, Verhulsts asserts that by reproduction, the population will grow proportionally to the current population. However, limiting causes such as starvation means the population will decrease at a rate proportional to the theoretical maximum capacity of the environment and the current population.

Verhulst's Logistic Model of Population 1 *Where $N(t)$ is the population at interval t, and r is the intrinsic growth rate and K number of individuals in equilibrium.*

$$\frac{dN(t)}{dt} = rN(t)(1 - \frac{N(t)}{K})$$

This creates a number of different possibilities in the nature of a population.

For $0 < r < 1$, The population will eventual diminish and die out, regardless of x_0

For $1 < r < 2$, The population will quickly stabilize at $\frac{r-1}{r}$.

For $2 < r < 3$, The population will oscillate around the same value, stabilizing at a linear rate

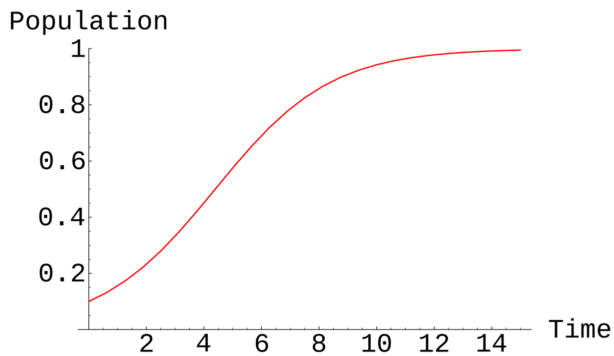

Figure 2: Logistic Representation of Population

Verhulst differed from Malthus in the fact that while he was equally or even more so politically motivated, he continuously opposed the use of mathematical models and theory to determine sociological and ethical judgement. This is in contrast to Malthus's use of theory to promote moral behavior and his successors use to promote the change of sexual behavior.

An interesting corollary of relevance is the biological train of thought termed r/K selection theory that was born from Verhulst's equation. This philosophy divides the reproduction and natural selection of species into two classes. R-selected species produce many offspring, each of which is unlikely to survive to adulthood. K-selected species invest more heavily in fewer offspring, each of which has a better chance of surviving to adulthood. Biologists find that members of each group share a large number of behavior patterns.

The logistic pattern is fairly useful and accurate when K is accurate. Verhulst predicted the upper limit of the Belgium population would be 9 400 000, only 900 000 off from the current populations of the country.

"While the Pearl-Reed logistic growth curve is no longer regarded as an accurate predictor of the growth over time of human populations, it was an important milestone in the development of demographic theory"[3]

There is certainly a degree of historical relevance in the study of Verhulst's method, however, none of these ideas would have such importance without the works of his successors. Most of the responsibility for the success of the Verhulst model of logistic population growth was actually the result of Raymond Pearl and Lowell J. Reed's groundbreaking work[8] in 1920 which modeled the growth of the United States population since 1790 with a high degree of precision.

Pearl and Reed first show the remarkable accuracy *a priori* of the geometric growth model for a period of 110 yearsleading up to the 1920 census. Using several data sets, they find only one instance where a Malthusian curve had beyond an error of one percent when predicting the 1920 population. Despite this impeccable success, Perl and Reed reject the long term stability of their models and echo the sentiment of Malthus and Verhulst that population growth is not infinitely stable when they come to the prediction that in the year 3000 the population of the United States will exceed 11.8 billion.

Pearl and Reed first obtained their logistic curve by defining several general properties.

1. Asymptotic to a line $y = k$ when $x = +\infty$
2. Asymptotic to a line $y = 0$ when $x = -\infty$
3. A point of inflection at some point $x = \alpha$ and $y = \beta$
4. Concave upwards to the left of $x = \alpha$ and concave downwards to the right of $x = \alpha$
5. No horizontal slope except at $x = \pm$
6. Values of y varying continuously from 0 to k as x varies from $-\infty$ to $+\infty$

Perl and Reed created the graph in Figure 3 under the beliefs that the United States had a carrying capacity of 200 million. So as the population at the time of the article was 100 million, the authors believed that they had reached the point of reflection in the logistic curve. As a point of interest, included is Figure 4, a model of current population growth.

Over time, the mathematical models of population dynamics evolved to deal with more complexity. The Lotka-Volterra model of population dynamics standalone falls within the realm of

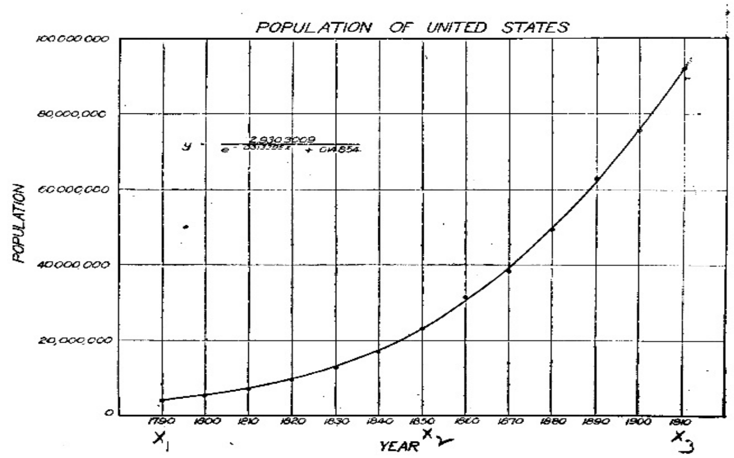

Figure 3: Model of United States Population Growth

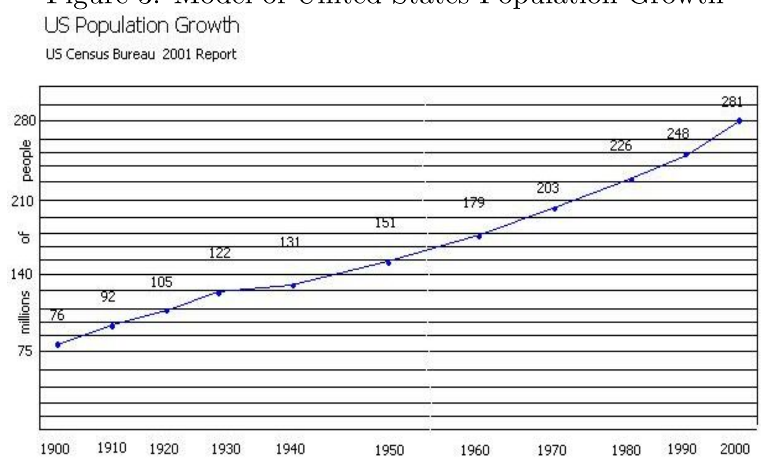

Figure 4: Model of United States Population Growth

being insignificant, what makes it important is that it is the general basis for most current models of animal interaction. So, while for this instance the model is inapplicable as we are generally concentrating on human populations, a brief outline is helpful.

Lotka-Volterra Model 1 *Describes the interaction of two species, where:*

$N_1 =$ *prey population*

$N_2 =$ *predator population*

$c =$ *efficiency of predators*

$r =$ *reproductive growth rate of* N_1

$d =$ *death rate for* N_2

$a =$ *conversion from prey to predator population*

$$\frac{dN_1}{dt} = rN_1 - cN_1N_2$$

$$\frac{dN_2}{dt} = acN_1N_2 - dN_2$$

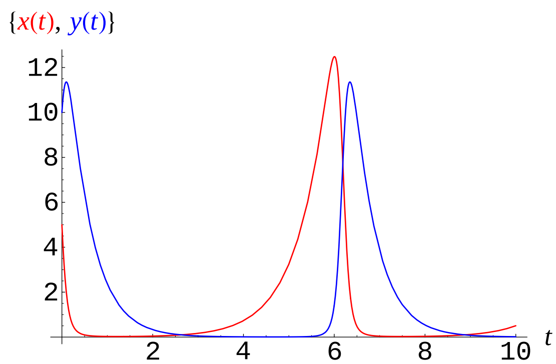

Figure 5: Lotka-Volterra Model of Predator/Prey Populations

The Lotka-Volterra model incorporates the fact that species tend not to live in a vacuum lacking interaction with other species. This divides the interactions into positive, mutually beneficial encounters and negative, predatory type interactions. The Lotka-Volterra model ends up repeating itself cyclically, the population of the prey will increase with the predator population lagging slightly until it hits its saturation point and decreases, with the predator population dying off after. This is a problem as it is a phenomena that is rarely observed in nature.

So again, as we saw with the Malthusian and Logistic models, while the overall equation of the model may not be true, the theory behind the model is what is most important.

The ability to forecast population changes has the ability to create political turmoil now. PBS and the show Nightline[6] at one point last year forecasted that the population of Arabs in Israel and the occupied territories within few generation will outnumber Israeli Jews. This prediction has lead to substantial movement towards two state solutions. If this were true it should be pretty obvious by comparing the r of the Arab and Israeli populations to determine if and when the Arab population will become dominant.

The problem with this report and Nightlines became the use of improper data. The Palestinian

Authoritys questionable counting practice resulted in more than a million nonexistent people. To compensate, they artificially inflated the birth rate numbers. This skewed the importance of the situation.

Assuming that there is no limits on growth on other side, the actual growth of the Palestinian population, while still faster than the Israeli growth is not fast enough to be a significant threat to Jewish dominance of Israeli and it is actually slowing down. In hard numbers, Israeli Jews have a x_n of around 5.5 million with a growth rate r of .0118, while the Arab Israeli and Palestian population is at 2.5 million (estimated due to lack of credible numbers) with a growth rate of around .0306 (again, suspect number).

Despite the appearance of a solid mathematical foundation, there is ample evidence that the extreme lifestyle changes that have occurred across the world have led of very large missteps in the calculation of population dynamics. At no time in history has man isolated himself from the Darwinist natural pressures that our evolutionary ancestors directly face, reaching into even the domain of god to push back the limits of nature.

This shifting demographic has caused a large number of egregious errors in population estimation. In 1951, the United Nations predicted that the population of the Earth would lie between $2.976X10^9$ and $3.636X10^9$. In reality, the population during 1980 was estimated at $4.45X10^9$. The United Nations in 1986 by estimated the world population for 2000 to hit 6 billion, when in reality, this was reached by 1997.

In a paper edited by Pearl, a founder of modern demographics, the environmental capacity limit was originally estimated to be 2 billion, a limit surpassed only four years later. Pearl changed this limit in 1936 to 2.6 billion, a number passed by 1955. Even in his milestone work in 1920, some of Pearl's assumptions were later rejected.

> *"...this is manifestly ridiculous; it would mean a population density of ... 3968 persons per square mile."*

Even in the United States this 'ridiculous' population density is easily exceeded by at least 15 population centers[1]. When attention is focused overseas, we see countries like Hong Kong which reach 16000 people per square mile!

An article[5] in the journal Technological Forecasting and Social Change provides for one of the most comprehensive reviews of the hurdles encountered in population modeling. A broad problem encountered is inability of researchers to accurately forecast the fluctuations in birth rates.

> *"Doubtless the most important source of error in population forecasts is uncertainty about future [fertility] rates, because these rates are changing over time in ways that so far have been difficult to predict or even to explain after the fact"*[2]

In fact there are numerous specific causes outside of reliable biological instinct which have enormous impact in birth rates. The average American tends to have an awkwardly high amount of awareness to the reproductive politics and impact on many undeveloped Asian and African countries, most namely India. These problems are not foreign to the history of the West either, however. Historically, birthrates have fluctuated based on the role of child. Until the Industrial Revolution, farming families were thought of as an asset of use in farm activities, while urban populations had a much lower birthrate generally because children were more of an economic burden than benefit. The Industrial Revolution induce population burst was not entirely the result of better living conditions or a general increase in economic status, but the shift of children to being useful assets by child labor. As a result, just as the farmers, the more children you had, the more income was available, especially when the importance of the health of the children was minimized.

A less abstract example occurs as the result of the French Revolution. Under the French monarchy, all property was inherited by the first born child. This was abolished by Napoleon code, which dictated that assets were divided equally by all children. Almost immediately the birth rate of the higher class approached one son per family. To the French aristocrats, the division of assets was an unacceptable surrender of pure social status.

Currently in France, over a period of nearly 30 years, fertility, measured by the total fertility rate, fell sharply, dropping from nearly 3 children per woman in 1964 to 1.65 in 1994. The list of European countries below replacement rate is not a lonely one, joining France are Ireland (1.99), Norway (1.81), Sweden (1.75), UK (1.74), Netherlands (1.73), Germany (1.37), Italy (1.33), Spain (1.32), and Greece (1.29).

To look ahead, what is the likely result of infanticide (often manifested in the form of abortion) as the result of one-child policies and the spread of prenatal genetic tests for sex? In India, one study came to the shocking conclusion that out of 8,000 selected abortions that occurred after genetic tests, only 3 were male fetuses. Where will fertility rate move to once the full impact of industrialization in India and China is fully realized?

Perhaps it is the greatest testament to the cleverness of man that he is conscious enough of the global implications of his footprint by mathematical deduction that he can attempt to reshape the natural order. While the methods enumerated were out of date, researchers still have just as much difficulty forecasting human nature. What we find is that the change in human behavior, especially in reproduction, is just as much directly influenced by politics and economics as it is by genetics. Until the world reaches some type of utopian level of economic and political stability, population dynamics will only be useful as a benchmark and not an exact measurement of the future.

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